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Estimation of Stoke's Parameters from Yale Polarimeter Data

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I

METHOD OF DATA REDUCTION

A. Recorder Output

The Jupiter polarimeter data has been recorded on six-channel Edin recorder charts. The proper orientation of the charts is such that the center of curvature of the vertical lines lies to the left(Fig. 1). The channels are labeled as follows, from bottom to top: (1) "R" (records total right hand plus one half total unpolarized power); (2) "L" (total left hand plus one half total unpolarized power); (3) "RL_1C" (cross correlation of left and right polarization vectors multiplied by the cosine of the relative phase of the channel - explanation below); (4) "RL_1S" (same as (3) except multiplied by sine instead of cosine); (5) and (6) "RL_2C" and "RL_2S", respectively (same as (3) and (4) but at an adjacent frequency. Not used in the present analysis). The first four channels are at 22.2 MHZ. Time runs right to left.

Between Jupiter storms the recoerder is run usually at a speed of 0.1 millimeters per second, and occasionally at 0.03 mm per second, with a time constant of 1.0 second. Calibrations occur every hour and are visible on all six channels as a series of five steps. They are followed a few minutes later by a series of four steps in the first two channels of somewhat greater amplitude. These are not to be confused with the calibration steps. Their purpose is to check the linearity of the detectors. Shortly before a storm are pen length checks which must be noted before attempting to reduce pulses. When a storm begins the speed is increased to 5.0 mm/sec and the time constant reduced to 0.1 sec. (Fig. 2)

B. Cross Correlation Channels

Channel 3 and 4 record, respectively, the quantity $RL_1\cos(\varphi_c+\theta)$ and $RL_2\sin(\varphi_c+\theta)$ in units of milliamps, where φ is the relative phase of the channel and $\theta = 2\chi$ where χ is the relative orientation of the ~~major axis of~~ the polarization ellipse. (For deails see: Marshall H. Cohen, Radio Astronomy Polarization Measurements, Proc. I.R.E., 46, 172,1958). Figure 3 contains a simplified diagram of the circuit. The signal $(RL/4)\cos[\varphi+\Delta\omega_\circ t]$ splits and each half mixes with a half of the reference signal $\cos[\psi-\Delta\omega_\circ t]$, one half of which is shifted ±90° in the phase by the "$k(\pi/2)$" phase box. Figure 4 contains a circuit diagram of the LSR phase shift unit, labeled "$m(\pi/2)$", which exchanges channel phases between steps 3 and 4 of the calibration sequence, assuming the channels to be in quadrature.

C. Calibration

The calibration steps are produced by current steps of approximatey 3.75, 7.5, 15.0, and 30.0 milliamps. 15.0 ma is used for step 3 and 4. Actual values have been recorded on each observing night in the observers' log book. Calibration curves are drawn for the first two channels simply by plotting the mean values of each step in millimeters relative to the base-line against the current for each step. The raw readings of channel 3 and 4 are to be corrected for phase by dividing by $\cos\varphi_c$ and $\sin\varphi_s$, respectively. The phases are determined by taking the ratio of the raw readings of step 3 and 4. For example consider channel 3. Taking absolute values only, step 3 is $RL_1\cos[\varphi_c \pm 90°] = RL_1\sin\varphi_c$, and step 4 is $RL_1\cos\varphi_c$. The ratio is

$$\frac{RL_1\sin\varphi_c}{RL_1\cos\varphi_c} = \tan\varphi_c,$$

from which φ_c is determined. Channel 4 is treated the same way. If linearity of the detectors is assumed calibration coefficients for all four channels may be determined by taking the slope of the best fit straight line which passes through the origin. (In practice it has been more feasible to weight the last steps more than the first.).

The uncertainty in measuring a calibration step can be estimated by dividing one third the peak to peak noise by the square root of the number of time constants in the step. For example if the peak to peak noise in a step is about 1.5 mm, $\tau = 1$ sec and the step lasts 30 seconds, $\sigma = 0.5/\sqrt{30}$ mm. The uncertainties are represented by error bars in the calibration curves. In the case of channel 3 and 4 the final error is the result of propagating 2 measuring errors of the above type. The uncertainty in a calibration coefficient is the difference in slope between the upper and lower curves (Fig. 5)

D. Reduction of Jupiter Pulses

Cohen had shown, for right and left circular antennas,

$$I = I_L + I_r \tag{1}$$

$$Q = 2RL\cos 2\chi \tag{2}$$

$$U = 2RL\sin 2\chi \tag{3}$$

$$V = I_L - I_R \tag{4}$$

In terms of the Stokes Parameters I, Q, U, V

$$m^2 = \frac{Q^2 + U^2 + V^2}{I^2} \tag{5}$$

and

$$\tan 2\beta = \frac{V}{\sqrt{Q^2 + U^2}} \tag{6}$$

from which

$$\tan \beta = r \qquad (7)$$

where m and r are fractional polarization and axial ratio, respectively. Thus

$$m^2 = \frac{(2RL)^2 + (I_L - I_R)^2}{I^2} \qquad (8)$$

$$\tan 2\beta = \frac{I_L - I_R}{2RL} \qquad (9)$$

Processing the data involves dividing a pulse into three sections: (1) baseline, (2) pulse, and (3) baseline. In practice it has been best to determine the divisions using channel 2, and then mark the corresponding divisions in the other channels after checking the difference in pen lengths. The data is then read off in equal time intervals (0.1 - 0.2 sec) relative to some arbitrary baseline <u>without</u> eyeball integration. Instantaneous readins are recorded from channels 1 to 4 at the same instant of time regardless of the noise, after checking the relative pen lengths carefully. Section 1 and 3 determine the true baseline by means of a least square straight line fit. Then the computer determines the deflections relative to the true baseline of all the points. Once the true baseline is established the computer determines the mean square noise in section 1 and 3, and upon applying the calibration coefficients, comes up with quantities σ_i^2 $(i = 1,4)$, which are noise corrections for the corresponding channels.

In theory if one reads a deflection d' which differs on account of noise from the true, or mean deflection d by an amount Δ, then

$$d' = d \pm \Delta.$$

Consider n measurements:

$$\begin{aligned} d'_1 &= d_1 \pm \Delta_1 \\ d'_2 &= d_2 \pm \Delta_2 \\ &\vdots \\ d'_i &= d_i \pm \Delta_i \\ &\vdots \\ d'_n &= d_n \pm \Delta_n. \end{aligned}$$

Adding the equations gives

$$\sum_1^n d'_i = \sum_1^n d_i + \sum_1^n (\pm\Delta_i) \qquad (i = 1,2,\ldots,n)$$

For n large

$$\sum_1^n (\pm\Delta_i) \to 0.$$

and

$$\sum_1^n d_i' = \sum_1^n d_i$$

or

$$\overline{d'} = \overline{d} \qquad \text{as to be expected.}$$

Now consider what happens when you square the equation $d_i' = d_i \pm \Delta_i$. For n measurements.

$$d_1'^2 = d_1^2 \pm 2d_1\Delta_1 + \Delta_1^2$$

$$\vdots$$

$$d_n'^2 = d_n^2 \pm 2d_n\Delta_n + \Delta_n^2.$$

For n large

$$\sum_1^n (\pm 2d_i\Delta_i) \to 0,$$

and

$$\sum_1^n d_i'^2 = \sum_1^n d_i^2 + \sum_1^n \Delta_i^2.$$

Dividing by n gives

$$\overline{d'^2} = \overline{d^2} + \overline{\Delta^2}$$

or

$$\overline{d^2} = \overline{d'^2} - \overline{\Delta^2}.$$

Thus if we wish to compute $(I_L + I_R)^2$, we write

$$I^2 = (I_L + I_R)^2 = I_L{}^2 + I_R{}^2 + 2I_LI_R$$

where

$$I_R = \frac{d_1 \pm \Delta_1}{c_i}, \text{ etc.} \qquad (c_i = \text{calibration coeff.})$$

Then

$$I_R^2 = \frac{d_1^2 \pm 2d_1\Delta_1 + \Delta_1^2}{c_1^2} \longrightarrow \frac{d_1^2}{c_1^2} + \frac{\Delta_1^2}{c_1^2},$$

where

$$\frac{\Delta_1^2}{c_1^2} = \sigma_1^2.$$

But,

$$2I_RI_L = 2\frac{(d_2 \pm \Delta_2)}{c_2}\frac{(d_1 \pm \Delta_1)}{c_1}$$

$$= \frac{2}{c_1c_2}(d_2d_1 \pm d_1\Delta_2 \pm d_2\Delta_1 \pm \Delta_2\Delta_1)$$

$$\longrightarrow 2d_2d_1/c_2c_1 \qquad (\Delta\text{s uncorrelated}).$$

Therefore

$$I^2 = I_L^2 + I_R^2 + 2I_LI_R + \sigma_1^2 + \sigma_2^2$$

and

$$I^2_{(\text{corrected})} = I^2 - \sigma_1^2 - \sigma_1^2.$$

Similarly

$$(I_L - I_R)^2_{\text{corrected}} = (I_L - I_R)^2 - \sigma_1^2 - \sigma_2^2.$$

Equation (8) and (9) involve a term RL which may be derived from the measured quantities

$$d_3 = c_3 RL\cos(\varphi_c + \theta) \qquad (10a)$$

$$d_4 = c_4 RL\sin(\varphi_s + \theta) \qquad (10b)$$

where $\varphi_c \neq \varphi_s$ and θ is the phase ~~introduced~~ angle introduced by Jupiter. Expanding (10) gives

$$d_3 = c_3 RL[\cos\varphi_c \cos\theta - \sin\varphi_c \sin\theta] \qquad (11a)$$

$$d_4 = c_4 RL[\sin\varphi_s \cos\theta + \cos\varphi_s \sin\theta], \qquad (11b)$$

followed by

$$\frac{d_3}{c_3 \sin\varphi_c} = RL[\cot\varphi_c \cos\theta - \sin\theta] \qquad (12a)$$

$$\frac{d_4}{c_4 \cos\varphi_s} = RL[\tan\varphi_s \cos\theta + \sin\theta]. \qquad (12b)$$

Adding gives

$$\frac{d_3}{c_3 \sin\varphi_c} + \frac{d_4}{c_4 \cos\varphi_s} = RL\cos\theta[\cot\varphi_c + \tan\varphi_s \qquad (13)$$

and

$$RL\cos\theta = \frac{(d_3/c_3)\cos\varphi_s + (d_4/c_4)\sin\varphi_c}{\sin\varphi_c \cos\varphi_s (\cot\varphi_c + \tan\varphi_s)}$$

$$= \frac{(d_3/c_3)\cos\varphi_s + (d_4/c_4)\sin\varphi_c}{\cos(\varphi_s - \varphi_c)}. \qquad (14)$$

In similar fashion

$$RL\sin\theta = \frac{(d_4/c_4)\cos\varphi_c - (d_3/c_3)\sin\varphi_s}{\cos(\varphi_s - \varphi_c)}. \qquad (15)$$

Squaring (14) and (15) gives

$$(RL\cos\theta)^2 = \frac{(d_3/c_3)^2\cos^2\varphi_s + 2(d_3/c_3)(d_4/c_4)\cos\varphi_s\sin\varphi_c + (d_4/c_4)^2\sin^2\varphi_c}{\cos^2(\varphi_c - \varphi_s)} \qquad (16)$$

and

$$(RL\sin\theta)^2 = \frac{(d_4/c_4)^2\cos^2\varphi_c - 2(d_3/c_3)(d_4/c_4)\sin\varphi_s\cos\varphi_c + (d_3/c_3)^2\sin^2\varphi_s}{\cos^2(\varphi_c - \varphi_s)} . \qquad (17)$$

Combining (16) and (17):

$$(RL)^2 = \frac{(d_3/c_3)^2 + (d_4/c_4)^2 + 2(d_3/c_3)(d_4/c_4)\sin(\varphi_c - \varphi_s)}{\cos^2(\varphi_c - \varphi_s)} . \qquad (18)$$

Equation (18) contains a term in the numerator involving $\sin(\varphi_c - \varphi_s)$, whose sign is determined by noting how the sign changes in going from calibration step 2 to 3, and by the fact that the phase switching unit adds 90° to one channel and subtracts it from the other. It is not known which channel has 90° added to it and which has it subtracted, but this does not matter. For example suppose it is observed that in going from step 2 to 3 the cosine channel goes from plus to minus and the sine channel from minus to plus. (1) If 90° is added to the sine channel and subtracted from the cosine channel, φ_c and φ_s both lie in the fourth quadrant only. (2) If the reverse is true φ_c and φ_s lie in the first and third quadrants, respectively. It can be seen that both (1) and (2) lead to the same sign of the argument $(\varphi_c - \varphi_s)$.

The absolute value and sign of $(\varphi_c - \varphi_s)$ can be checked by observing changes in Faraday rotation with time. Places where the deflections in the cosine and sine channels vary in sign in a reasonably periodic fashion , one can estimate the difference $\varphi_c - \varphi_s$ and its sign according to which channel lags the other. For $\varphi_c - \varphi_s = 0$ the channels are in quadrature and a maximum of one corresponds to a null of the other.

For each time coordinate x there corresponds one data card containing five numbers: d_1, d_2, d_3, d_4, and x, where $d_1,\ldots,$ etc. are the four deflections and x is the time coordinate (1, 2, 3,..., etc.). Each number is alloted five spaces such that the first 25 spaces are used (program statement 10= FORMAT(5F5.1)). Each section of cards is separated by a card containing -70.0 in the first five spaces, and each block of cards (= 3 sections) is separated by a card containing the same number. The statement "DO 1000 KKK = 1,N" determines how many blocks of data the computer will process, where N equals the number of blocks.

The computer output contains statistical scatter corresponding to the noise in the channels, but does not take into account the uncertainties in the calibrations nor the argument $(\varphi_c - \varphi_s)$.

II

SUMMARY OF DATA REDUCTION: 1/24/67 - 6/12/67

Until 2/22/67 important information about the phase shift unit was lacking, such as how the phase shift was introduced and whether or not it was accurately 90°. Therefore, it was necessary, starting on 2/6/67, to investigate whether or not the calibration coefficients c_3 and c_4 could be determined by means of periodic changes in Faraday rotation with time.

If a maximum of each channel is measured, the deflections are

$$d_{3max} = c_3 RL\cos 0° \tag{19a}$$
$$d_{4max} = c_4 RL\sin 90° \tag{19b}$$

and

$$c_3/c_4 = \frac{d_{3max}}{d_{4max}} = \rho$$

At some calibration current we measure

$$d_3 = c\rho RL\cos \varphi_c \tag{20a}$$

$$d_4 = cRL\sin \varphi_s \tag{20b}$$

Setting $\varphi_c = \varphi_s + \Delta\varphi$, expanding $\cos(\varphi_s + \Delta\varphi)$, and dividing (20a) by (20b) gives

$$\frac{1}{\rho}\frac{d_3}{d_4} = \frac{\cos \varphi_s \cos \Delta\varphi - \sin \varphi_s \sin \Delta\varphi}{\sin \varphi_s}$$

$$= \cot \varphi_s \cos \Delta\varphi - \sin \Delta\varphi,$$

or

$$\cot \varphi_s = \frac{(1/\rho)(d_3/d_4) + \sin \Delta\varphi}{\cos \Delta\varphi}, \tag{21}$$

where $\Delta\varphi$ may be determined ideally by measuring the fraction of a period a maximum of one channel differs in time from a null of the other. φ_s is then obtained from (21). Finally, c_3 and c_4 are obtained by rearranging (20a) and (20b):

$$c_3 = c\rho = \frac{d_3}{RL\cos \varphi_c} \tag{22a}$$

$$c_4 = c = \frac{d_4}{RL\sin \varphi_s}, \tag{22b}$$

where RL is the calibration current.

But, it was found that this method gave such large uncertainties in

(φ_c - φ_s) as to be practically useless. The followed an investigation as to what effect an error in th 90° phase insertion would have on the calibration coefficients, and m and r. On 2/22/67 the phase shift unit was located, untampered with, at Balcones Research Center, and its properties and reliability were at last established.

Prior to the first serious attempt at data reduction an investigation was done on the stability of the channel gains over a period of seven hours. It was found that channel 1 and 2 were approximately linear up to 30 ma input current, and that the largest excursions in gain were on the order of 8% and 4%, respectively. Channel 3 and 4 showed possible non-linearity, but not in excess of the limits of uncertainty inmeasurement. Excursions were also approximately within the limits of uncertainty of approximately 12% and 16%, respectively. Then followed some hand reductions of peaks of pulses in the 11/18-19/65 Jupiter storms which showed significant fluctuations, but the error analysis revealed that it was not possible to attribute these fluctuations in m and r to anything other than equipment noise.

Beginning at the end of March and continuing through May 1967 a new method of reduction was developed. It was evident from the previous work that quantitative values of m and r were uncertain by 50% - 100%, due to measuring uncertainties of the pulses and calibrations. Therefore it was decided to terminate any further attempts at obtaining absolute values of m and r, and to concentrate on detecting inherent changes in the polarization parameters. The technique involves reading instantaneous, unsmoothed deflections at equal time intervals of .1 or .2 seconds. The data is then fed into a computer which prints out values of I^2, $(I_L - I_R)2$, $(RL)^2$, m^2 and r^2. The advantages of this method are: (1) The uncertainty in the calibration coefficients can be eliminated, thereby reducing the statistical uncertainty in m and r to that attributable to the noise in the channels. (2) Time consuming erroranalysis can be eliminated; errors can be estimated adequately from the statistical fluctuations in the output. (3) Inherent changes in m and r larger than the standard deviation of the statistical fluctuations can be detected in time intervals as short as about 0.5 second. This information is practically as important as absolute values themselves, for in the case of m, an inherent change implies that m is <u>not</u> always unity. The assumption is kept in mind, of course, that the channel gains are constant during time intervals up to a few minutes, but may drift during a period of many minutes to an hour by 10 or 15 percent.

So far, the best data reduced by the new technique occurs on 11/27/65 at about 7:10 EST. Graphs of m and r versus t are in <u>figure 6</u> for two relatively long pulses starting at 7:09.6 and ending at 7:10.9. The average value of m in the 7:09.6 graph appears to be between .95 and 1.00; that for the 7:10.6 graph about .90. This slight difference of 5% - 10% can probably be accounted for by small errors in baseline determination. The slight negative slope of each graph can likewise be attributed to small baseline slopes relative to the true baseline. There appear to be small variations of $\Delta m = .20 - .25$ with periods of 2 - 3 seconds, but since the standard deviation is of the same order of magnitude, one should not conclude with confidence that these are inherent. One useful aspect of the apparent constancy of m, however, is that the detection system appears to be linear for a considerably higher antenna temperature than that of the strongest calibration step, owing to the small probability that the effect on m due to nonlinearity would be exactly canceled by an inherent change in m. Hence, it would be fair to state that channel 2 is linear to a current of 150 milliamperes above background.

At 7:09.6 r appears essentially constant over a period of 10 seconds, although small fluctuations of $\Delta r \approx 0.1$ show up in the interval t = 2.0 - 5.0 sec., which, owing to less scatter, give better evidence that propagation effects are being detected than in the case of m. Similar fluctuations occur in the 7:10.4 graph with the addition of a slight rise in the middle.

Some data from the 11/18/65 and 11/19/65 Jupiter storms were processed by an earlier program which did not contain the noise corrections. Nearly all the graphs of m have so much scatter as to be practically useless. A plot of averages of m of each pulse in the Nov. 19th data with error bars representing scatter indicates there may be a change occuring at about 7:30 EST (Fig. 7). Histograms (Fig. 8) for the time intervals 7:10 - 7:30 and 7:30 - 7:50 show peaks at m = .70 and 1.10, respectively. However, the possibility of a change in channel gain occuring in this period could preclude interpreting the variation in m as inherent. The rest of the output suffers from too much scatter and too short a time duration.

III

CONCLUSION AND RECOMMENDATIONS

Up to this point there has not been much evidence that the fractional polarization changes from unity. It is clear that the chief problem with the Yale polarimeter data is the noise, especially in the cosine and sine channels. Moreover, statistical fluctuations in RL carry extra weight because of the factor of 4 in the m equation and 2 in the tan 2β equation. It prevents being able to detect small changes in m and r with certainty, and prevents precise absolute measurements from being made.

The noise of all channels should in the future be kept at a minimum. Increasing the time constant to 0.2 sec. may be helpful without appreciable loss in time structure. Many calibrations have been degraded or completely obliterated by disturbances other than pure noise. Observers should keep a check on the tracings at all times. If it appears that the calibrations are not coming through clearly before a storm, they should be applied manually at some quieter moment. It is important, moreover, to make sure that at least one clear calibration sequence occurs immediately <u>after</u> a storm so that one may obtain calibration coefficients during the storm by interpolation. Finally, the calibration steps should be larger so that one does not have to worry about extrapolating into the nonlinear region of the detectors. Furthermore, increased step size and reduction of noise should make it possible to read the calibrations during a quiet period of a storm with great accuracy.

Recently, it has been found that a more valid treatment of the data is to take readings every tenth of a second, rather than two tenths. This is to insure that the points are properly distributed around the mean.

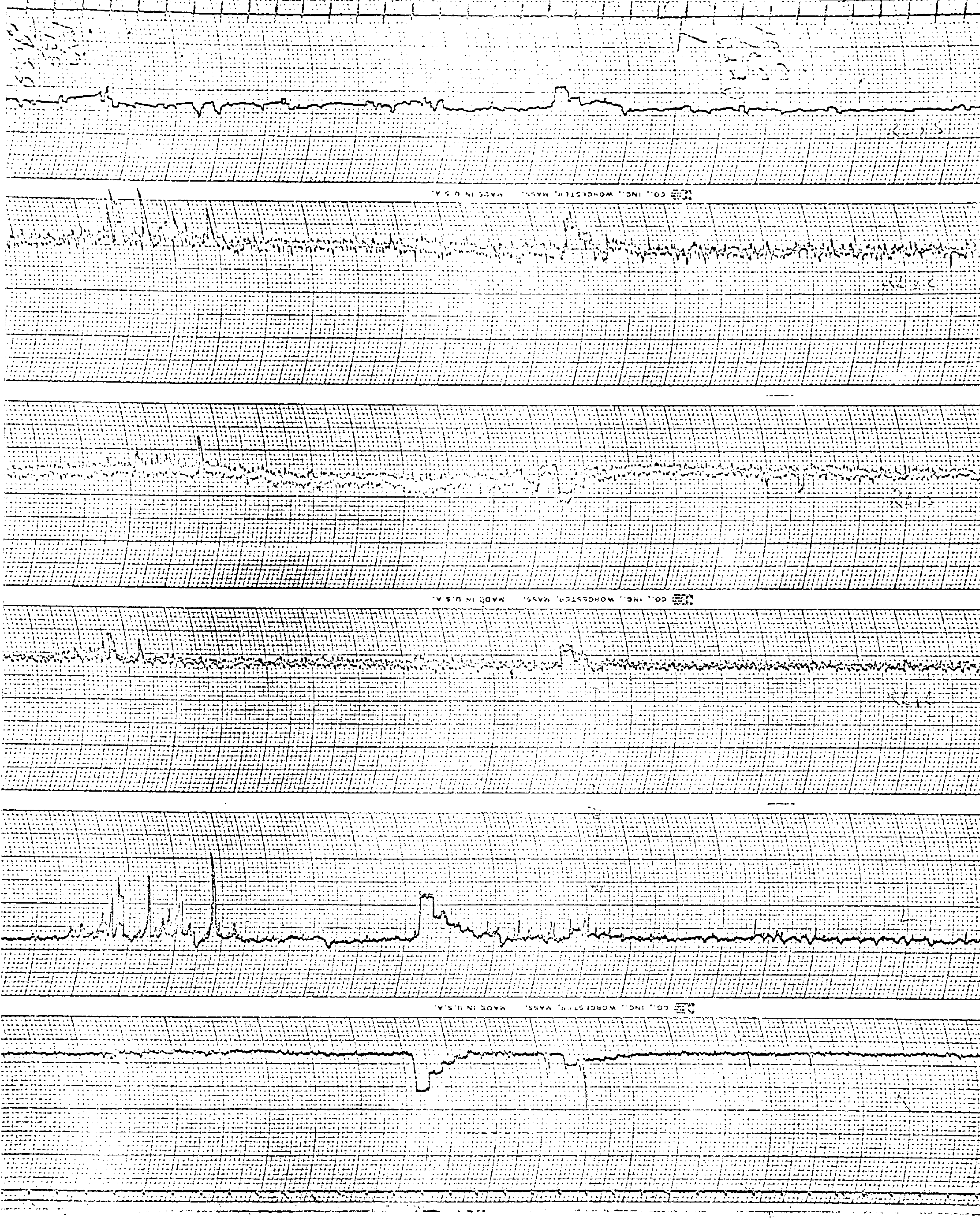

Fig. 1 - Calibrations

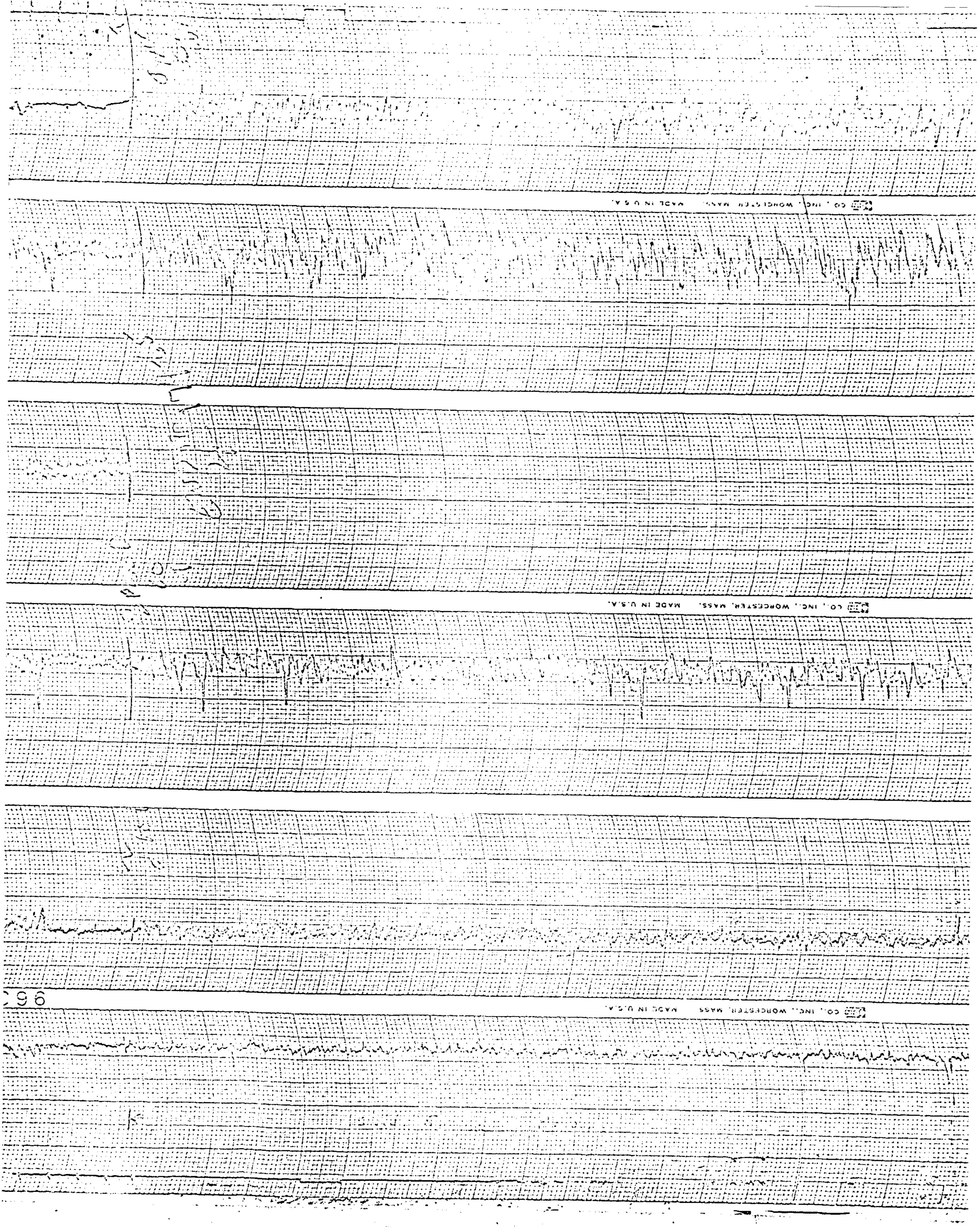

Fig. 2. End of Jupiter storm

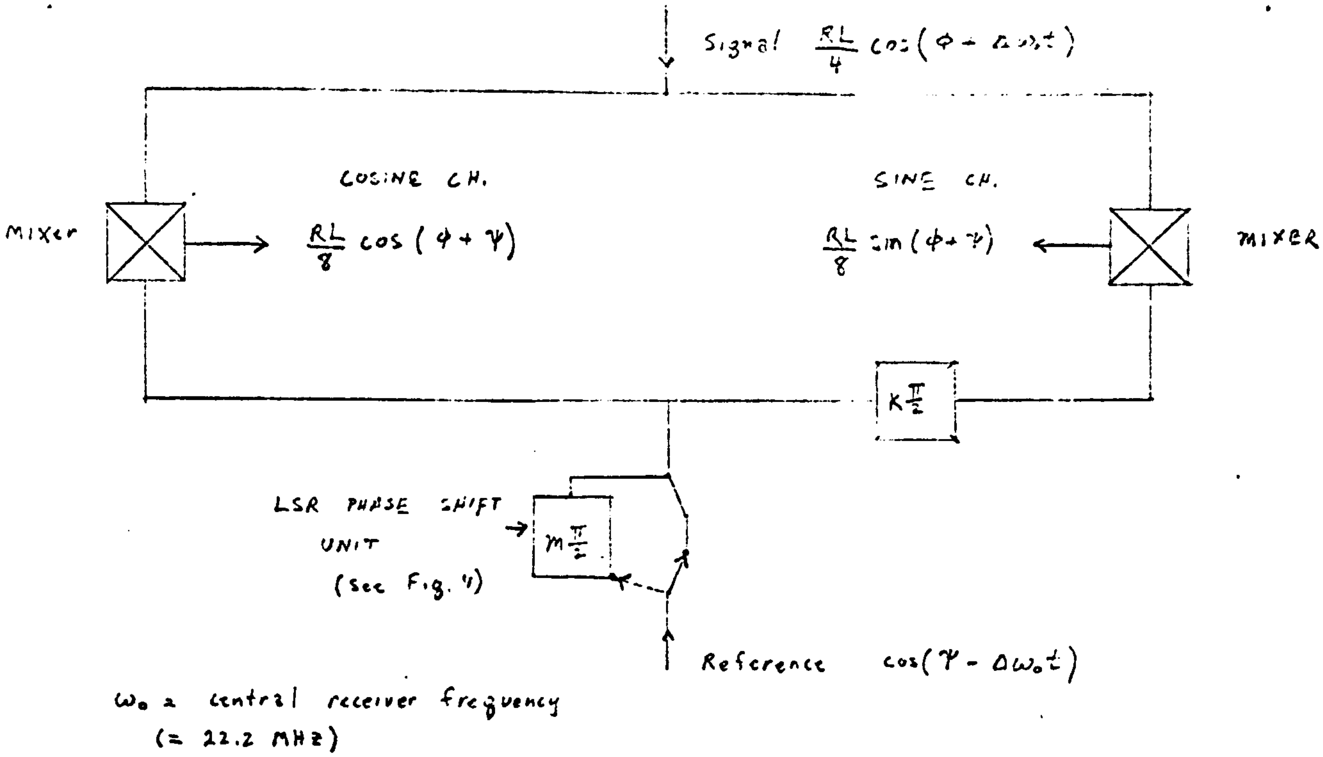

Fig. 3 Cross Correlator Circuit

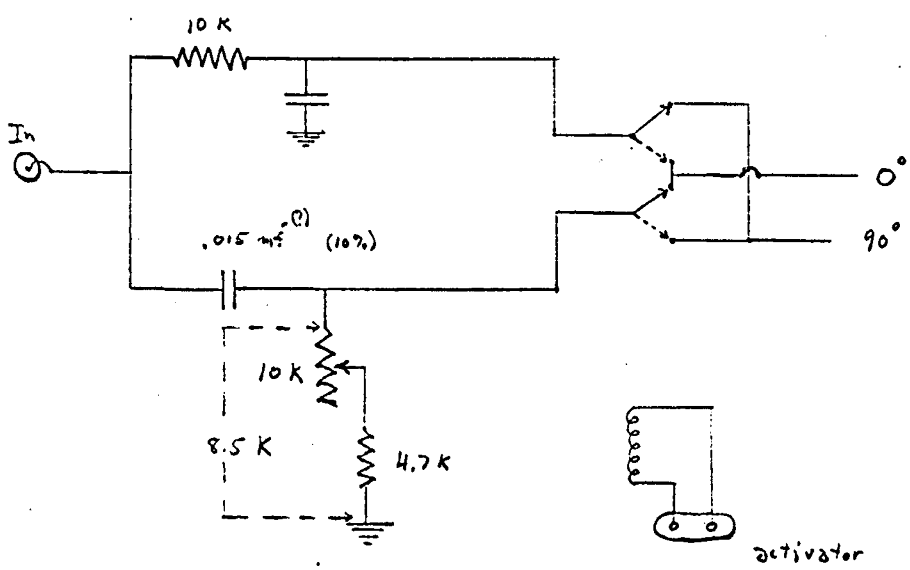

Fig. 4 LSR Phase Shift Unit

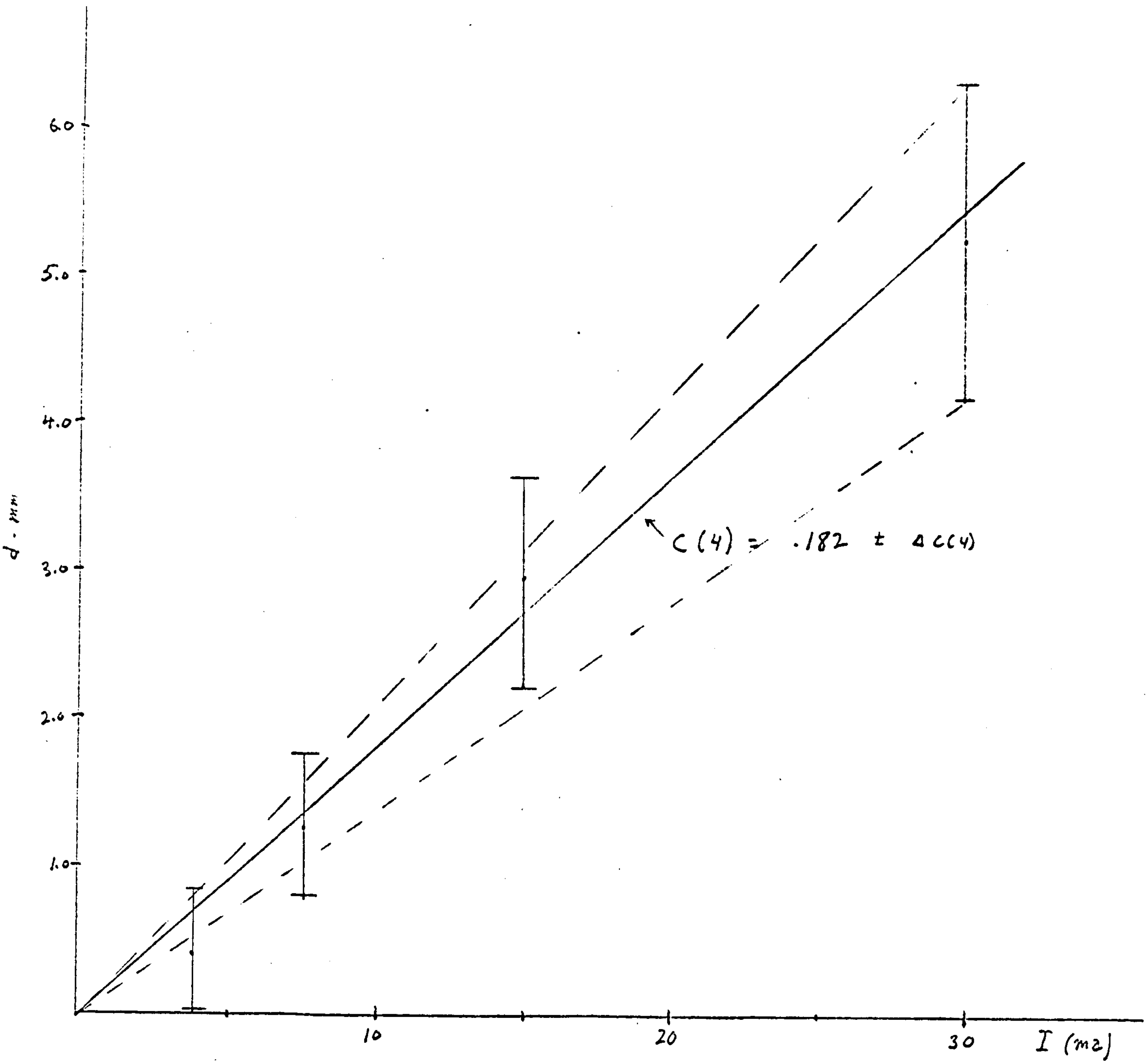

FIG. 5 SAMPLE CALIBRATION CURVE

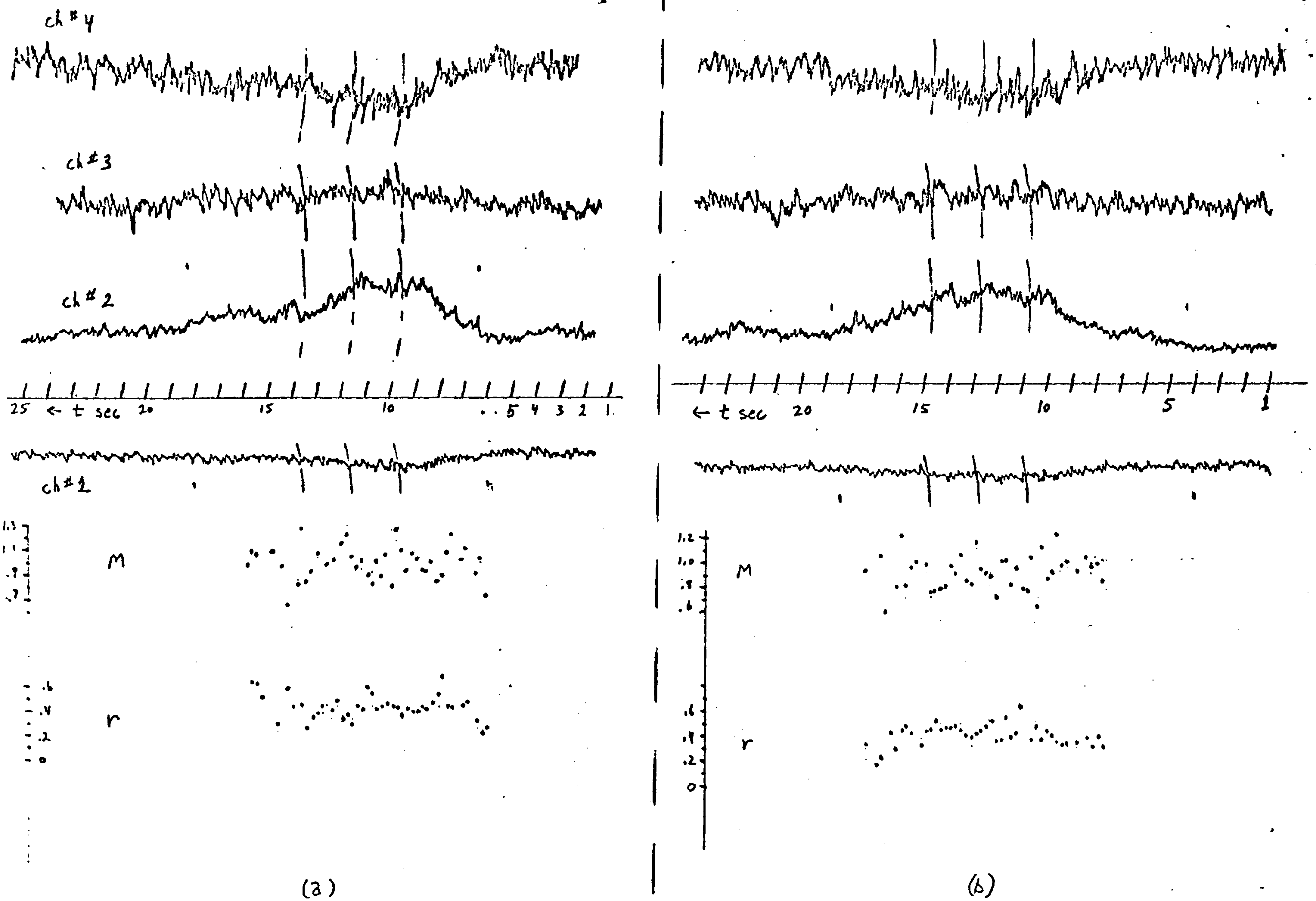

FIG. 6

Jupiter pulses in 11/27/65 data at (a) 7:09.[illegible] EST, and (b) 7:10.4 EST with corresponding plots of M and r.

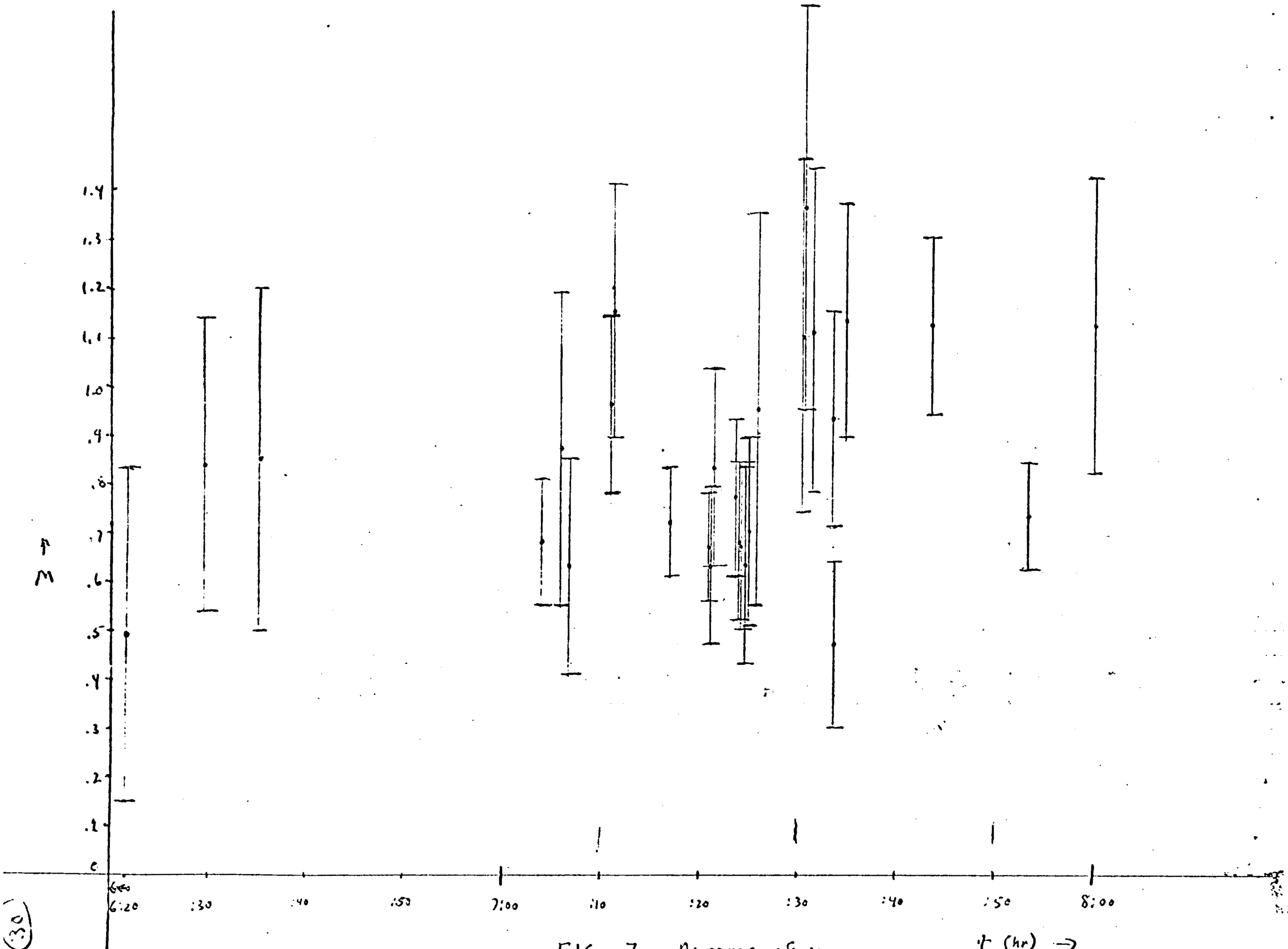

FIG. 7 – Averages of m

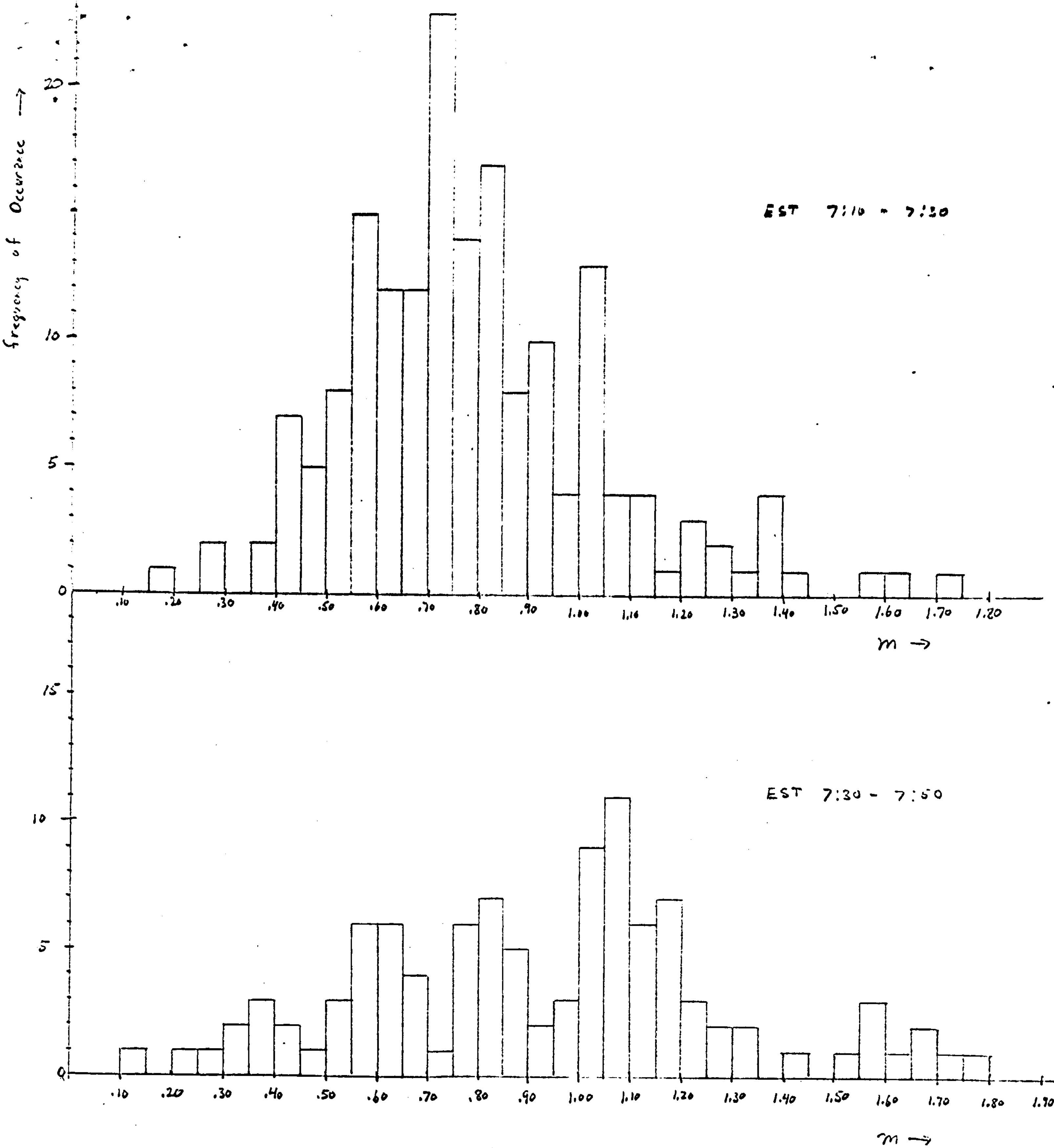

Fig. 8 Histograms for 11/19/65 data